# Back paper Examination

Physics III,
B. Math., $3^{rd}$ year,
September - December 2022.
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December $26^{th}$, 2022, Morning Session.
Duration: 3 hours.
Total points: **80**

Please give arguments where necessary. If it is unclear from your answer why a particular step is being taken, full credit will not be awarded. Grades will be awarded not only based on what final answer you get, but also on the intermediate steps.

1. **7+7+(3+3)=20**

   An infinitely large (in all three directions) linear dielectric material with dielectric constant $\epsilon_a$ carries a uniform electric field $\vec{E}_0$. A cylindrical rod (infinite in length along the axis) with a circular cross-section of radius $R$ which is built out of a second linear dielectric material of dielectric constant $\epsilon_b$ is immersed in the first. $\vec{E}_0$ is oriented transverse to the cylindrical axis.

   (a) Find the electric potential inside and outside the sphere.

   (b) Find the electric field inside and outside the sphere.

   (c) Schematically, draw the electric field lines throughout space for

   - $\epsilon_a < \epsilon_b$
   - $\epsilon_a > \epsilon_b$

2. **(2+3)+5+10+5 = 25**

   (a) In certain time-independent situations, a magnetic field can be obtained from a magnetic scalar potential $\psi_M$ instead of the vector potential $\vec{A}$, such that $\vec{B} = -\vec{\nabla}\psi_M$. This is very advantageous for computation of the magnetic field, since one deals with a scalar field than a vector field.

   - Under what condition (other than time-independence) can such a magnetic scalar potential be defined?
   - Show that the magnetic scalar potential always satisfies the Laplace's equation.

(b) Imagine an infinite wire carrying a uniform current $I$. From what you know about the magnetic field generated by such a current, find the scalar potential everywhere it can be unambiguously defined, at least up to a constant.

(c) The Coulomb gauge condition guarantees that in the Cartesian co-ordinate system, a current density along a Cartesian axis produces a vector potential along that axis alone. This is not generally true in the spherical and/or the cylindrical co-ordinate system. Show that in the case of an azimuthally symmetric current, $\vec{j}(\vec{r}) = j(r,\theta)\hat{e}_\phi$, we get $\vec{A}(\vec{r}) = A(r,\theta)\hat{e}_\phi$ in the Coulomb gauge.

(d) As mentioned in the previous part, under the Coulomb gauge condition,

$$\vec{A}(\vec{r}) = \frac{\mu_0}{4\pi}\int_{\vec{r}_1} \frac{\vec{j}(\vec{r}_1)}{|\vec{r}-\vec{r}_1|} d\mathbb{V}_1$$

Find a gauge such that the magnetic vector potential can be represented as

$$\vec{A}(\vec{r}) = \frac{1}{4\pi}\int_{\vec{r}_1} \frac{\vec{B}(\vec{r}_1)\times(\vec{r}-\vec{r}_1)}{|\vec{r}-\vec{r}_1|^3} d\mathbb{V}_1$$

(*Hint: you may want to think about laws of magnetostatics that you already know, instead of searching in the whole gauge function space*)

3. **3 + (5 + 5 + 2) = 15** In lectures, we discussed the advantages of working with the fictitious *magnetic charges* $\rho_M$ and $\sigma_M$ in the presence of magnetization but no free current.

(a) Express the total dipole moment of a magnetized body of finite volume $\mathbb{V}$ with magnetization $\vec{M}(\vec{r})$ in terms of the magnetic charge $\rho_M$ and other geometric factors and as an integral over the volume.

(b) The half-space $z > 0$ has uniform magnetization $\vec{M}(\vec{r}) = -M\hat{e}_z$. The half-space $z < 0$ has magnetization $\vec{M}(\vec{r}) = +M\hat{e}_z$. Do not assume simple magnetic matter. Find the magnetic field everywhere

i. Using the induced current densities.

ii. Using the induced magnetic charge densities.

iii. Verify that the two approaches give the same result for the magnetic field.

4. **(5+1)+(6+6+2) = 20**

(a) An infinite wire, extended along the y - axis, is located at $z = h$. It carries a current $I$. Lying in the $x - y$ plane and with opposite pairs of sides parallel to the $x-$ and the $y-$axes respectively, a square loop of side $b$ moves with a velocity $\vec{v} = v\hat{e}_x$. Find the magnitude of the EMF induced in the loop the moment the centre of the loop intersects the $z$-axis. Given that the direction of the current is along

$+\hat{e}_y$, is the EMF clockwise or anti-clockwise when seen from above the $x - y$ plane? Explain your answer.

(b) An infinite solenoid of radius $b$ has $n$ turns per unit length. The current in the solenoid varies in time as $I = I_0 \cos(\omega t)$. A ring with radius $a < b$ is placed in the solenoid perpendicular to the axis, its axis coinciding with the solenoidal axis. Assume the ring is Ohmic with resistance $R$.

- Find the current induced in the ring, along with its direction (w.r.t to the direction of the solenoidal current).
- Find the maximum magnitude of the force an infinitesimal piece of the ring of length $\vec{dl}$ experiences. At what values of $t$ does this magnitude occur?
- Find the net force on the ring at all times.